

Asteroseismology with solar-like oscillations

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Abstract. Almost 100 years ago Sir Arthur Eddington noted that the interiors of stars were inaccessible to observations. The advent of helio- and asteroseismology has completely changed this assessment. Helioseismology has provided very detailed information about the interior structure and dynamics of the Sun, highlighting remaining issues in our understanding of the solar interior. In the last decade extensive observations of stellar oscillations, in particular from space photometry, have provided very detailed information about the global and internal properties of stars. Here I provide an overview of these developments, including the remarkable insight that has been obtained on the properties of evolved stars.

1 Introduction

In his seminal book *The internal constitution of the stars* Eddington (1926) pondered ‘What appliance can pierce through the outer layers of a star and test the conditions within?’. He answered the question through his theoretical investigations of stellar interiors which, despite the limited physical knowledge at the time, were remarkably successful in uncovering the basic principles underlying stellar internal structure. Modelling stellar structure and evolution has undoubtedly become much more sophisticated and, presumably, realistic since Eddington’s time, but the basic question remains: which *observations* can pierce through the outer layers of a star and test the conditions within? The answer is now well-established: the study of stellar interiors through the observation of oscillations on their surface, in other words, *asteroseismology*.

The detailed observational study of stellar interiors started with the development of *helioseismology*, from extensive observations of oscillations on the solar surface (for a review, see, e.g., Christensen-Dalsgaard 2002, see also Kosovichev, this volume). However, as indicated in Fig. 1 oscillations are found in a broad range of stars, providing opportunities for studies of stars in essentially all phases of their evolution. In this brief review I focus on stars showing oscillations similar to those of the Sun which, as discussed below, are intrinsically stable and excited stochastically by the near-surface convection.

Modes of solar-like oscillations are generally characterized by extremely small amplitudes, in the solar case up to about 20 cm s^{-1} in radial velocity and a few parts per million in intensity. The difficult observations of such oscillations in distant stars had a modest beginning in the nineties (e.g., Brown et al. 1991;

Kjeldsen et al. 1995; Bouchy & Carrier 2001), but they have evolved dramatically in the last few years through space-based photometric observations from the CoRoT (Baglin et al. 2013) and, in particular, the NASA *Kepler* mission (Borucki et al. 2010; Gilliland et al. 2010) launched in March 2009. Thus we stand at the beginning of a new phase of strongly observationally constrained studies of stellar interiors.

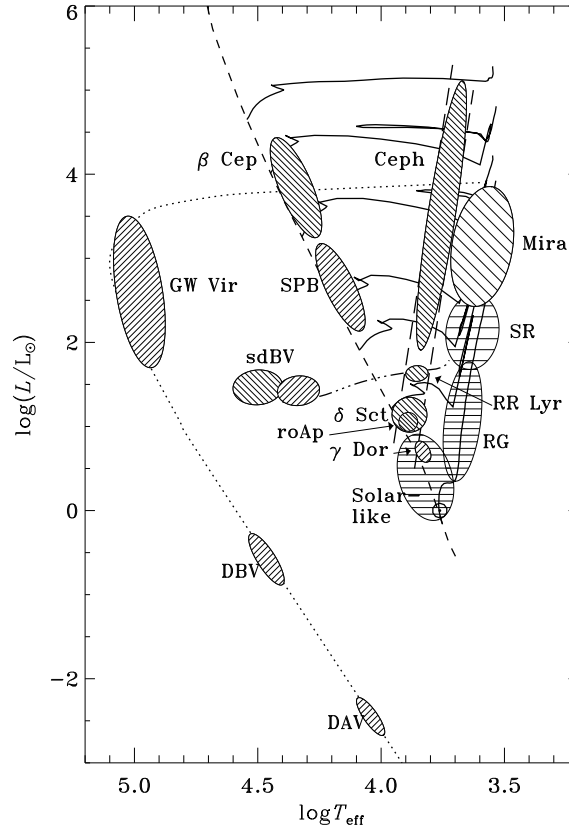


Fig. 1. Hertzsprung-Russell diagram showing the location of various groups of pulsating stars. The dashed line shows the zero-age main sequence and the solid lines show selected evolutionary tracks. The dotted line schematically indicates the white-dwarf cooling track. Here I focus on solar-like pulsators, indicated by horizontal hatching and situated to the right of the Cepheid instability strip (marked ‘Ceph’). For further details, see Aerts et al. (2010).

2 Basic properties of stellar oscillations

Here I only give a brief overview of the main features of stellar oscillations. For a detailed description, see Aerts et al. (2010).

Stellar oscillations are characterized by the dominant restoring forces and the mechanisms exciting the modes. Another important characteristic is the geometrical structure of the mode. In (nearly) spherically symmetric stars this is characterized by a spherical harmonic $Y_l^m(\theta, \phi)$ as a function of co-latitude θ and longitude ϕ . Here the degree l measures the total number of nodal lines on the stellar surface and the azimuthal order m , with $|m| \leq l$, gives the number of nodal lines crossing the equator. Spherically symmetric modes, with $l = 0$, are known as radial oscillations. In addition, a mode is characterized by the radial order n related, sometimes in a rather complex manner (Takata 2012), to the number of nodes in the radial direction. In distant stars, where only oscillations in light integrated over the stellar surface have so far been analysed, cancellation suppresses the modes of higher degree, and only modes up to typically $l = 2 - 3$ are observed.

The two dominant restoring forces are pressure, and buoyancy acting on density differences. The pressure-driven modes, known as p modes, are essentially standing sound waves. These are the modes observed in the Sun and solar-like oscillations in moderately evolved stars. They tend to have relatively high frequency, in the solar case between 1000 and 5000 μHz , corresponding to periods between 17 and 3 minutes, and high radial order. Consequently, their properties are well characterized by their asymptotic behaviour, according to which, to leading order, the cyclic frequency satisfies

$$\nu_{nl} \simeq \Delta\nu \left(n + \frac{l}{2} + \epsilon \right), \quad (1)$$

where ϵ is a frequency-dependent phase largely determined by the near-surface properties. Here the *large frequency separation* is given by

$$\Delta\nu = \left(2 \int_0^r \frac{dr}{c} \right)^{-1}, \quad (2)$$

where c is the adiabatic sound speed and the integral is over distance r to the centre, between the centre and the surface radius R . It may be shown that $\Delta\nu$, and hence the frequencies, scales as the square root of the stellar mean density, $\Delta\nu \propto (M/R^3)^{1/2}$ (Ulrich 1986), where M is the mass of the star.

The departure from the simple relation (1) contains important diagnostic information. This is characterized by the *small frequency separation*

$$\delta\nu_{nl} = \nu_{nl} - \nu_{n-1, l+2} \simeq -(4l+6) \frac{\Delta\nu}{4\pi^2\nu_{nl}} \int_0^R \frac{dc}{dr} \frac{dr}{r}, \quad (3)$$

where the last expression is valid only for main-sequence stars. Here the integral is weighted towards the centre and hence is sensitive to the sound-speed structure

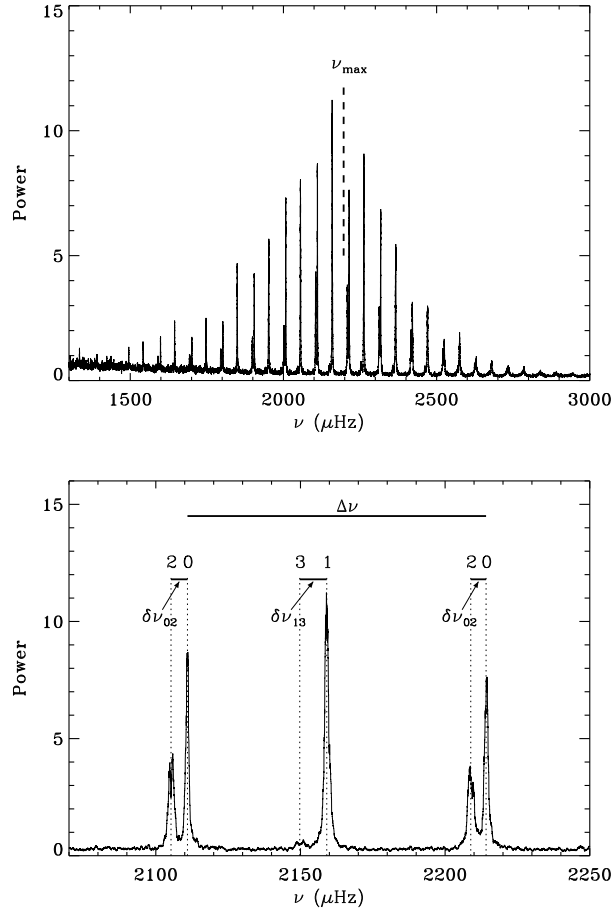


Fig. 2. Power spectrum of the solar-like pulsator 16 Cygni A, from 35 months of *Kepler* data, smoothed with a $0.6 \mu\text{Hz}$ boxcar filter. The lower panel shows a short segment; the degrees of the modes and the large and small frequency separations are indicated. The inferred frequency ν_{max} in the top panel, marked by a vertical dashed line, and the fitted frequencies, marked by vertical dotted lines in the lower panel, were obtained by Lund et al. (in preparation).

in the core. For an approximately ideal gas, $c^2 \propto T/\mu$, where T is temperature and μ is the mean molecular weight; therefore $\delta\nu_{nl}$ is sensitive to the composition of the core, and hence to the amount of hydrogen that has been converted to helium by nuclear fusion, determined by the age of the star. As noted by e.g. Christensen-Dalsgaard (1988) this provides a simply way to determine stellar ages, if other properties of the star are known.

The properties of the acoustic-mode spectrum in main-sequence stars are illustrated in Fig. 2 showing the power spectrum of the star 16 Cygni A as observed by *Kepler*. The lower panel identifies the large and small frequency separations, based on frequency fits to the power which in this case allows detection of modes of degree up to 3.

The second restoring force is gravity, acting through buoyancy on density differences across horizontal surfaces; consequently, this only operates for $l > 0$. The resulting modes are standing internal gravity waves, or g modes. They are characterized by the so-called buoyancy, or Brunt-Väisälä, frequency N , given by

$$N^2 = g \left(\frac{1}{\Gamma_1} \frac{d \ln p}{dr} - \frac{d \ln \rho}{dr} \right), \quad (4)$$

where g is the local gravitational acceleration, p is pressure, ρ is density and Γ_1 is the adiabatic compressibility. In convection zones N^2 is negative, and hence the gravity waves are evanescent. In main-sequence stars the g modes have relatively low frequency, and their detection in the Sun has been hotly debated for decades (García et al. 2007; Appourchaux et al. 2010). However, in evolved stars the gravitational acceleration, and hence N^2 , gets very high in the compact core of the stars, and hence g modes may have high frequency, in the range of the solar-like p modes. This gives rise to the very interesting phenomena of *mixed modes* to which we return below.

As for p modes, the relevant g modes are often of high radial order, making their asymptotic behaviour of great diagnostic value. This is most simply expressed in terms of the oscillation period $\Pi = 1/\nu$ which approximately satisfies

$$\Pi_{nl} = \Delta \Pi_l (n + \epsilon_g), \quad (5)$$

where ϵ_g is a phase that may depend on the degree. Here the period spacing is

$$\Delta \Pi_l = \frac{2\pi^2}{[l(l+1)]^{1/2}} \left(\int N \frac{dr}{r} \right)^{-1}. \quad (6)$$

For spherically symmetric stars the frequencies are independent of the azimuthal order m . This degeneracy is broken by departures from spherical symmetry, of which by far the most important is rotation. Rotation gives rise to a splitting which, for slow rotation, can be written as

$$\nu_{nlm} = \nu_{nl0} + m \delta_{\text{rot}} \nu_{nlm} \quad (7)$$

where $\delta_{\text{rot}} \nu_{nlm}$ reflects an average of the internal rotation rate, weighted by the properties of the oscillations. In the solar case, this has allowed a detailed determination of solar internal rotation (for a review, see Howe 2009). In the stellar case less information is obviously available, but, as discussed below, some very interesting results have been obtained.

Solar-like oscillations are intrinsically damped but gain their energy from the acoustic noise generated by the near-surface convection. The result are peaks in the power spectrum with an amplitude that is determined by the

balance between the energy input and the damping and a width that, for sufficiently long observations, is determined by the intrinsic damping rate (e.g., Christensen-Dalsgaard et al. 1989). Early estimates of the energy input and the resulting amplitudes in the solar case were made by Goldreich & Keeley (1977), while Christensen-Dalsgaard & Frandsen (1983) made a first estimate of the amplitudes of solar-like oscillations across the relevant part of the Hertzsprung-Russell diagram. The damping is dominated by the effects of convection, involving the perturbations to both the convective heat flux and the turbulent pressure (e.g., Balmforth 1992). The treatment of these effects is highly uncertain, although various formulations of time-dependent convection have been established (see Houdek & Dupret 2015, for a review). With appropriate choice of parameters a reasonable fit can be obtained to the observed solar line widths (Chaplin et al. 2005; Houdek 2006).

The combined result of the excitation and damping is a characteristic distribution of power with frequency (Goldreich et al. 1994), as shown in Fig. 2 for the observations of 16 Cyg A. This is characterized by the frequency ν_{\max} at maximum power. There is substantial empirical evidence that ν_{\max} scales as the acoustic cut-off frequency in the stellar atmosphere (e.g., Brown et al. 1991; Stello et al. 2008), leading to $\nu_{\max} \propto MR^{-2}T_{\text{eff}}^{-1/2}$, where T_{eff} is the effective temperature. The physical reason for this scaling has not been definitely established, although Belkacem et al. (2011) pointed out some likely relevant factors.

3 Asteroseismic determination of stellar properties

The space-based asteroseismic observations from *CoroT* and *Kepler* have set the scene for extensive investigations of stellar properties, ranging from ensemble studies of large numbers of stars to detailed studies of individual targets. These missions were, in part, motivated by the study of extra-solar planetary systems (exoplanets) through the transit technique, with the common requirement with asteroseismology of very high photometric precision over long periods of time. We are still only at the beginning of exploring the potential of these asteroseismic data, and here I can just give a brief indication of the results obtained. A recent review of asteroseismology based on solar-like oscillations was provided by Chaplin & Miglio (2013).

The most basic observed properties of solar-like oscillations are the frequency ν_{\max} at maximum power and the large frequency separation $\Delta\nu$. These can be determined even from data with a low signal-to-noise ratio. Assuming that T_{eff} is determined independently, the scaling relations with acoustic cut-off frequency and stellar mean density then provide two equations which can be solved for the mass and radius (Kallinger et al. 2010). Even this simple analysis provides stellar quantities that are otherwise very difficult to determine. It can be refined by including constraints based on stellar model grids, including also information about the stellar composition (Gai et al. 2011). A detailed test of these techniques was carried out by Silva Aguirre et al. (2012). Alternatively, given the

somewhat shaky foundations of the scaling for ν_{\max} , fits to grids of models can be carried out just based on $\Delta\nu$ and T_{eff} (Lundkvist et al. 2014).

These techniques provide simple methods to determine the basic properties of large numbers of stars. They have been applied extensively to the CoRoT and *Kepler* observations, and only a few examples can be given here. Chaplin et al. (2011, 2014) analysed *Kepler* observations of substantial samples of main-sequence stars, in the early paper also comparing with the predicted distributions, from Galactic modelling, in mass and radii of stars in the solar neighbourhood. Data for huge numbers of red giants have been obtained by CoRoT and *Kepler*, allowing detailed characterization of the population of these stars (e.g., Hekker et al. 2011). Analysis of stars in open clusters is particularly interesting. Thus, based on *Kepler* data, Miglio et al. (2012) estimated the red-giant mass loss in two open clusters from determination of stellar masses in different evolutionary stages.

A very important application of basic asteroseismology of red giants is in *Galactic archaeology*, relating stellar properties to the location of the stars in the Galaxy (Miglio et al. 2009). For red giants there is a close relation between stellar mass and age, and hence just the simple asteroseismic analysis provides a measure of stellar age (Miglio et al. 2013). When combined with large-scale spectroscopic investigations this provides the basis for a detailed investigation of the chemical and dynamical evolution of the Galaxy (e.g., Casagrande et al. 2016).

When individual frequencies have been determined much more detailed and accurate investigations of stellar overall and internal properties are possible. A difficulty in such analyses is the uncertain treatment of the near-surface layers in the star and their effects on the oscillation frequencies, giving rise to what is known as the near-surface error in the computed frequencies. In the solar case this can be isolated in the analysis owing to the availability of observations over a large range of degrees. Various techniques have been developed to correct for the effect in distant stars, based on an assumed similarity with the solar correction (e.g., Kjeldsen et al. 2008; Christensen-Dalsgaard 2012) or with a somewhat stronger physical basis (Ball & Gizon 2014). Alternatively, model fits can be based on suitable ratios between small and large frequency separation which are largely insensitive to the near-surface effects (Roxburgh & Vorontsov 2003; Otí Floranes et al. 2005).

An early analysis of *Kepler* data was carried out by Metcalfe et al. (2010), for a star in the subgiant phase where hydrogen has been exhausted in the core. The resulting compact helium core increased the frequencies of gravity waves in the deep interior, giving rise to modes of mixed p- and g-mode nature. The frequencies of such modes are very sensitive to the internal properties of the star, including its age, and hence the fit to the observed frequencies in principle may result in very precise determinations of stellar properties. However, several solutions were in fact found, each tightly constrained by the data. This is an example of the importance of including additional information about the star, supplementing the asteroseismic data, as constraints on the stellar properties. A detailed analysis of a sample of stars observed by *Kepler* was carried out by

Mathur et al. (2012) who also obtained some information about the dependence of the surface correction on stellar properties. Extensive modelling of two stars observed by *Kepler* was carried out by Silva Aguirre et al. (2013), using several different modelling and fitting techniques to test the range of systematic uncertainties involved in such fits. Interestingly, one of the stars, with a mass of around $1.25 M_{\odot}$, had clear evidence for a convective core, with some additional mixing outside the unstable region. This is a first indication of the potential for using asteroseismology of solar-like stars to study the physics of stellar interiors. Metcalfe et al. (2015) fitted the full set of *Kepler* data for the two components of the binary star 16 Cygni (see also Fig. 2). As an encouraging test of consistency the independent analysis of the two components yielded the same age, around 7 Gyr, within the errors of 0.25 Gyr, in accordance with the assumption of contemporaneous formation of the pair.

Asteroseismology is playing an important role in the determination of properties of exoplanet host stars, benefitting from the fact that the same photometric observations can be used both for the characterization of the exoplanets and for asteroseismology. To determine the properties of an exoplanet we need the radius and mass of the host star which can be determined much more accurately with asteroseismology than with ‘classical’ astrophysical techniques. Furthermore, asteroseismology allows determination of the age of the host star and hence the planetary system. In this way Batalha et al. (2011) found that *Kepler*’s first rocky exoplanet, with a radius of $1.4 R_{\text{E}}$, orbited a star with an age of around 10 Gyr, twice the age of the Sun. Analysis of further data for this system by Fogtman-Schulz et al. (2014) yielded a value of the age of 10.4 ± 1.4 Gyr and, remarkably, allowed a determination of the radius of the planet with a precision of 125 km. A similar age was obtained by Campante et al. (2015) for a system containing 5 planets with sizes at or smaller than that of the Earth. These striking results demonstrate that planet formation took place already in the early phases of the history of the Galaxy. I also note that a detailed analysis of a CoRoT exoplanet host was carried out by Lebreton & Goupil (2014), who investigated the extent to which different combinations of seismic and non-seismic data could constrain the properties of the star.

Silva Aguirre et al. (2015) carried out a detailed analysis of the 33 *Kepler* confirmed or potential exoplanet host stars for which extensive asteroseismic data are available. Taking into account also systematic effects of the use of different modelling or fitting techniques, they were able to determine the radii and masses with median uncertainties of 1.2 and 3.3 per cent, respectively, whereas the ages were determined with a median uncertainty of 14 per cent. The distributions of uncertainties are shown in Fig. 3. I note that the age is determined predominantly from the decrease in the central hydrogen abundance; thus the fractional uncertainty in age, as illustrated, is unavoidably higher for unevolved stars.

As mentioned in Section 2 (see Eq. 7) rotation causes a splitting of the frequencies according to the azimuthal order m which in fact has been observed in a number of cases in the *Kepler* data. The resulting information about the

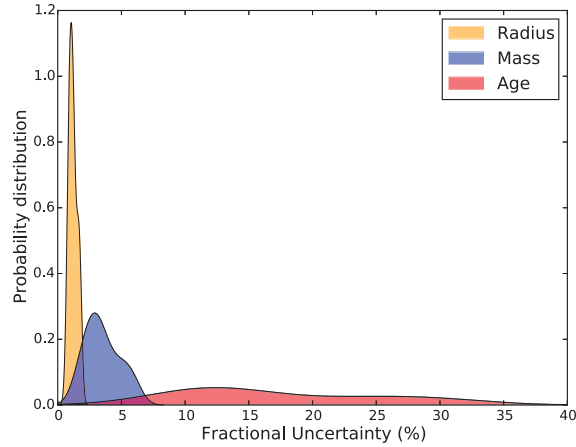


Fig. 3. Distributions of fractional uncertainties in asteroseismic determinations of radius, mass and age for 33 *Kepler* (potential) exoplanet hosts. Adapted from Silva Aguirre et al. (2015).

stellar rotation rate is obviously of substantial interest. However, in the exoplanet context an even more interesting aspect is information about the orientation of the rotation axis. For stochastically excited modes it is not unreasonable to assume that the average amplitude, for given n and l , is independent of m . However, the *observed* amplitude depends on the inclination of the rotation axis with respect to the line of sight (Gizon & Solanki 2004). In the limiting case of a rotation axis in the plane of the sky only modes with even $l - m$ are observed, while if the rotation axis points towards the observer only modes with $m = 0$ are seen. For exoplanets detected with the transit technique one would naively expect the rotation axis of the host star to be in the plane of the sky: the planets are assumed to form from a disk left over from the formation of an initially rapidly rotating star and hence lying in the star's equatorial plane; thus the rotation axis would be approximately orthogonal to the plane of the planetary orbits, as is indeed the case for the solar system. Such systems have indeed been found (e.g., Chaplin et al. 2013). However, in other cases there is a large misalignment between the rotation axis and the axis of the planetary orbits (e.g., Huber et al. 2013; Lund et al. 2014). Understanding the origin of this behaviour is an important part of the study of the formation and evolution of planetary systems.

4 Asteroseismology of red giants

As a background to the discussion of the asteroseismology of red giants it is useful to give a brief overview of red-giant evolution; for a detailed review, see

Salaris et al. (2002). This phase of stellar evolution follows after the end of central hydrogen burning. The star continues to obtain its energy from hydrogen fusion, but now in a shell around the gradually growing helium core. The core contracts while the outer layers expand and cool, establishing a deep outer convection zone. When the star reaches the Hayashi track the continuing expansion takes place at nearly constant effective temperature, leading to a drastic increase in the luminosity (see also Fig. 1), which in the solar case will reach as high as one thousand times the present luminosity, along the red-giant branch. At this point the temperature in the helium core has reached a level, around 100 million degrees, where helium fusion to carbon and oxygen sets in. The core expands and the outer layers contract, until the star settles down to a phase of quiescent helium burning, a substantial fraction of the energy still coming from the hydrogen shell burning. After the end of central helium burning the outer layers again expand greatly in the asymptotic giant phase, after which the star sheds its envelope and is left with the central very compact carbon-oxygen core, a white dwarf.

Given the deep outer convection zone it was expected (Christensen-Dalsgaard & Frandsen 1983) that red giants would show solar-like oscillations. The first detection was made by Frandsen et al. (2002), followed by a few other ground-based studies which, however, were hampered by the very long observation periods required to resolve the low frequencies resulting from the low mean density of the stars. However, as already mentioned, a major break-through in the study of solar-like oscillations in red giants came with the space-based observations from CoRoT and *Kepler* which have shown oscillations in tens of thousands of stars. The oscillations can be followed to the most luminous stars observed by *Kepler*, with a power envelope similar to what is observed on the main sequence (cf. Fig. 2) but with a dominant frequency less than $1\ \mu\text{Hz}$, corresponding to a period of more than 10 days (Stello et al. 2014). Indeed, the oscillations observed by *Kepler* merge with the even slower oscillations seen in highly evolved giants with ground-based surveys (Mosser et al. 2013), and there is evidence that semi-regular variables, with periods of many months and typically observed by amateur astronomers, show solar-like oscillations (Christensen-Dalsgaard et al. 2001).

From the first observations there were indications that the red-giant modes had a very short damping time, leading to broad peaks in the power spectrum and hence limited frequency precision, even though an early theoretical estimate indicated life times several times the typical solar values (Houdek & Gough 2002). Also, apparently only radial modes were observed. This would further limit the diagnostic value of the observations. A first indication of nonradial oscillations in a red giant was obtained by Hekker et al. (2006) from observations of line-profile variations. A definite proof that red giants showed the full range of solar-like oscillations was obtained by De Ridder et al. (2009) in an early analysis of CoRoT data which also demonstrated the similarity of the power envelope over a broad range of stellar luminosities and hence frequencies of maximum power.

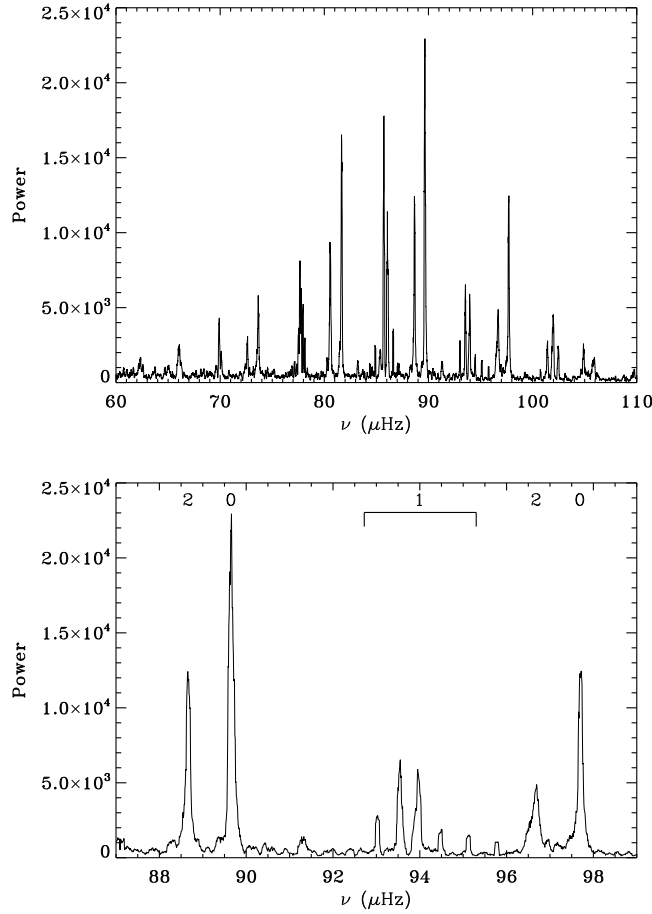


Fig. 4. Power spectrum from 49 months of *Kepler* observations of the red giant KIC6779699, smoothed with a $0.09 \mu\text{Hz}$ boxcar filter. In the small segment of the spectrum in the lower panel the degrees of the modes are indicated, the horizontal line marking the $l = 1$ mixed modes. See Bedding et al. (2011).

Although the detection of non-radial modes in solar-like oscillations of red giants was an important step, the full, huge diagnostic potential of these observations became apparent with the identification of mixed modes in a red giant by Beck et al. (2011), in *Kepler* observations. This was followed by studies of ensembles of red giants by Bedding et al. (2011) from *Kepler*, and Mosser et al. (2011) from CoRoT, observations. An example of an observed power spectrum is shown in Fig. 4. This is superficially similar (albeit at lower frequencies) to the main-sequence power spectrum in Fig. 2, with pairs of peaks of degree $l = 0$

and 2; but instead of a single intermediate $l = 1$ peak there is now a group of peaks; these are modes of mixed p- and g-mode behaviour.

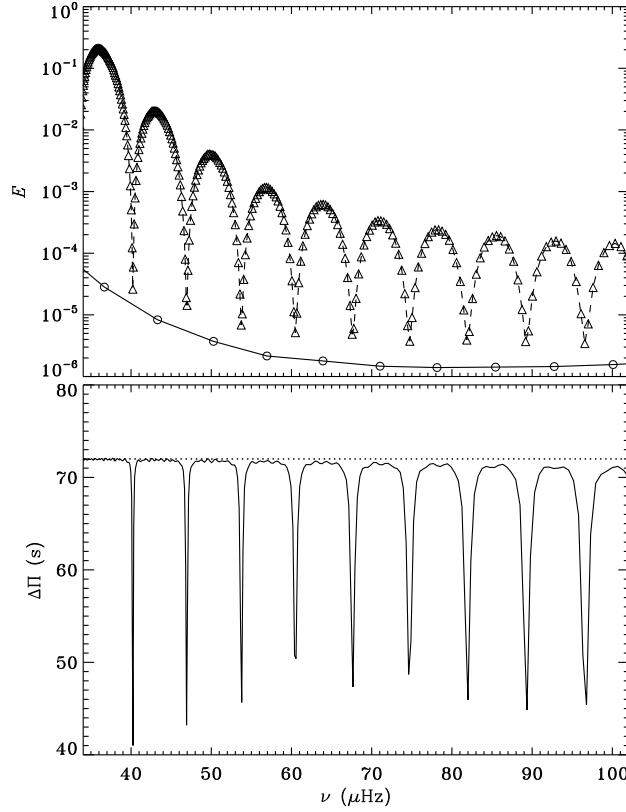


Fig. 5. Properties of oscillations in a red-giant model, of mass $1 M_{\odot}$ and radius $7 R_{\odot}$. The upper panel shows the normalized mode inertia (cf. Eq. 8) for modes of degree $l = 0$ (circles and solid line) and 1 (triangles and dashed line). The lower panel shows the computed period spacings for $l = 1$, the dotted horizontal line marking the asymptotic value (cf. Eq. 6).

To understand this behaviour it is instructive to consider the properties of modes in a stellar model, characterized in terms of the normalized mode inertia

$$E = \frac{\int_V \rho |\boldsymbol{\delta r}|^2 dV}{M |\boldsymbol{\delta r}|_{\text{phot}}^2}, \quad (8)$$

where the integral is over the volume of the star, $\boldsymbol{\delta r}$ is the displacement vector and $|\boldsymbol{\delta r}|_{\text{phot}}$ is its magnitude at the photosphere. With this normalization E is relatively small for modes trapped in the outer parts of the star, whereas E can

be very large for modes trapped in the deep interior. E is plotted in Fig. 5 as a function of frequency for modes of degree $l = 0$ and 1 in a red-giant model. The radial modes are purely acoustic and have a small inertia that generally decreases with increasing frequency. The $l = 1$ modes are generally predominantly g modes, trapped in the deep interior below the convective envelope, and hence have large inertia. However, there are acoustic resonances where the inertia decreases to values not much higher than the radial-mode inertia at the corresponding frequency. Here the modes have their largest amplitude in the envelope where they have an acoustic character. The location of these resonances, and the radial-mode frequencies, approximately satisfy the asymptotic relation in Eq. (1). Given that the processes exciting and damping the modes predominantly take place in the near-surface layers where the convective velocities are large, it is intuitively clear that modes with low inertia are easier to excite and hence are expected to be more visible in the power spectra of the observations (see Dupret et al. 2009; Grosjean et al. 2014). This is the origin of group of $l = 1$ peaks in Fig. 4; these are modes with inertia somewhat higher than the radial-mode inertia, but still excited to observable amplitudes.

Given the asymptotic behaviour of g modes (Eq. 5) the properties of the mixed modes are most naturally analysed in terms of period spacings which, as shown in the lower panel of Fig. 5, are also affected by the acoustic resonances. For the modes of predominantly g-mode character the spacing $\Delta\Pi = \Pi_{nl} - \Pi_{n-1l}$ is close to the asymptotic value (cf. Eq. 6) shown by the horizontal line. However, at the acoustic resonances the period spacing takes on a characteristic ‘V’-shape as a function of frequency, a behaviour that led Beck et al. (2011) to the first identification of mixed modes in red-giant observations.

From the observed frequencies of the mixed modes one can determine the period spacings around the acoustic resonances and, most reliably from a fit to the detailed asymptotic behaviour of the frequencies (Mosser et al. 2012a), determine the asymptotic period spacing $\Delta\Pi_l$ (Eq. 6). It was shown by Bedding et al. (2011) and Mosser et al. (2011) that the period spacing provides a clear separation between otherwise very similar stars ascending the red-giant branch with just shell hydrogen fusion and stars in the core helium-fusion phase: the period spacing was substantially smaller in the former case than in the latter. This can be understood from Eq. (6), according to which the asymptotic period spacing is determined by an integral over the buoyancy frequency (Eq. 4). When the star moves to the core helium-burning phase the core expands, and this decreases the local gravitational acceleration and hence the buoyancy frequency. A further reduction of the integral results from the convective core caused by helium fusion, since the integration in Eq. (6) excludes the convective core. Both effects decrease the magnitude of the integral and hence increase the asymptotic period spacing. Combining the period spacing with the large frequency separation $\Delta\nu$, which varies strongly with stellar radius, allows detailed diagnostics of stellar evolution, as discussed by Mosser et al. (2014).

Further information about stellar interior structure, such as the properties of the convective core in helium-burning stars, may in principle be obtained from

detailed fitting of the individual oscillation frequencies. Although such fits have been attempted in a few cases (e.g., Di Mauro et al. 2011; Jiang et al. 2011) much work is still required to explore these possibilities.

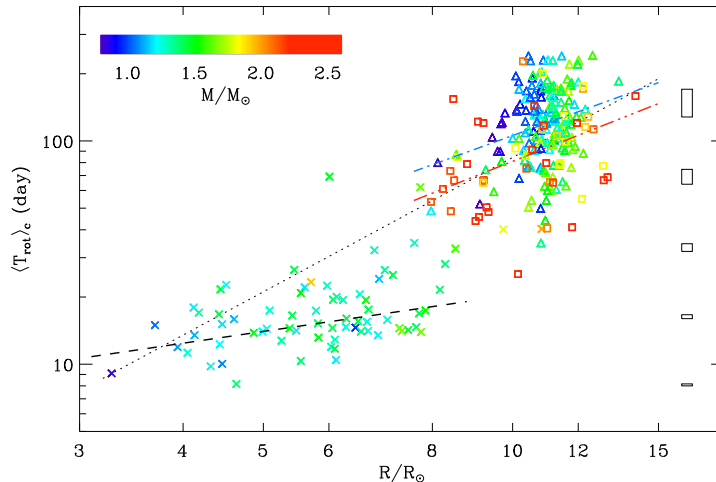


Fig. 6. Asteroseismically inferred core rotation periods in red giants (crosses) and core helium-burning stars (triangles and squares), plotted against stellar radius in solar units. The colour code indicates stellar mass. The right-hand boxes show typical errors, depending on the period. From Mosser et al. (2012b).

From a determination of the rotational splitting (cf. Eq. 7) in *Kepler* observations of a red giant Beck et al. (2012) concluded that the core of the star rotated faster than the surface by around a factor 10. This was based on determining the splitting for mixed modes, including modes with a substantial g-mode component where the splitting was dominated by the core. Fast core rotation was also found through asteroseismic inversion in less evolved stars, in the sub-giant phase and near the base of the red-giant branch, by Deheuvels et al. (2012, 2014) and Di Mauro et al. (2016). As shown in Fig. 6 Mosser et al. (2012b) determined the core rotation of a large number of stars on the red-giant branch and in the core helium-burning phase. Combined with the nearly uniform rotation inferred in the solar interior (cf. Howe 2009) and a recent asteroseismic determination of overall rotation in old field stars (van Saders et al. 2016) these results provide indications of the evolution of stellar interior rotation with age, a process that may also have important consequences for stellar structure evolution as a result of related instabilities and mixing processes.

In fact, the rapid core rotation in red giants should come as no surprise. As discussed above the evolution on the red-giant branch involves a strong contraction of the core. If there were local conservation of angular momentum this would result a spin-up of the core to far higher rotation rates on the red-giant

branch than in fact inferred from the asteroseismic determinations. Thus some angular-momentum transport mechanism must be operating in the stellar interior, leading to a reduction of the angular momentum and hence the rotation rate in the core. The normally considered mechanisms for angular-momentum transport in stellar interiors are somewhat uncertain. However, it has been found that they are insufficient, by one to two orders of magnitude, to account for the observed rotation in red-giant stars (e.g., Eggenberger et al. 2012; Marques et al. 2013; Cantiello et al. 2014). Thus additional transport mechanisms are required. Internal gravity waves (Fuller et al. 2014) or mixed modes (Belkacem et al. 2015) may play an important role. Even so, it is clear that we are still not close to understanding these important aspects of stellar evolution.

An early mystery in the study of solar-like oscillations in red giants was the suppression of the $l = 1$ modes in some stars with otherwise apparently normal oscillation spectra (Mosser et al. 2012; García et al. 2014). Stello et al. (2016) demonstrated that these stars had a mass, slightly higher than the Sun, such that they would have had convective cores on the main sequence. On this basis Fuller et al. (2015) proposed that the $l = 1$ modes were suppressed by scattering by a fossil magnetic field in the core of the star, generated through dynamo action when the star was on the main sequence. Although this model needs to be tested through more detailed calculations, it represents yet another instance of the power of asteroseismology to probe the evolution of these evolved stars.

5 Future prospects

The CoRoT mission ended operations in December 2013 after 7 years and the *Kepler* nominal mission ended in the spring of 2013 with the breakdown of two of its four reaction wheels. This, however, is far from the end of space asteroseismology. Operations of *Kepler* are continuing in the K2 mission, where successive fields along the Ecliptic are observed for three-months periods (Howell et al. 2014). With this orientation stable pointing of the satellite can be achieved with just the remaining two reaction wheels. Early results from this mode of operation are promising, both for asteroseismology of stars near the main sequence (Chaplin et al. 2015) and for red-giant observations as applied to Galactic archaeology (Stello et al. 2015). This will be followed by the TESS mission (Transiting Exoplanet Survey Satellite, Ricker et al. 2014) scheduled for launch by NASA in 2017. Over a two-year period TESS will make a survey of nearly the entire sky, to search for exoplanets and carry out asteroseismology. Most fields will be observed for around 28 days, but for two fields at the ecliptic poles the observations will be continuous for a year each. A major advantage of TESS compared with *Kepler* is the focus on relatively nearby stars which greatly enhances the possibilities for supplementary ground- and space-based observations. This advantage is shared by ESA’s PLATO mission (Rauer et al. 2014), selected for launch in 2024. PLATO will observe fields much larger than *Kepler*’s, in two cases for two or three years, emphasising the characterization of Earth-like planets in the habitable zone. Asteroseismology will be possible for a large fraction

of the exoplanet candidates detected and, obviously, for a large number of other stars.

In parallel with the exciting prospects offered by these new missions, the analysis of the CoRoT and nominal *Kepler* data has far from been completed. Indeed, efforts to go beyond the basic characteristics of the stars are just starting. An important example is the characterization of convective core overshoot in main-sequence stars from analysis of solar-like oscillations (Deheuvels et al. 2015a). The evolution of rotation with age will remain a key topic of research. Interestingly, from *Kepler* data Benomar et al. (2015) found a general tendency to near-uniform rotation in main-sequence stars, as has also been found in the Sun from helioseismology, and in strong contrast to the rapid core rotation on the red-giant branch. Moving to later evolutionary stages Deheuvels et al. (2015b) found little radial variation of rotation in core helium-burning stars. Thus we see the first signs of an overall characterization of rotation in the different evolutionary stages.

Despite the success of space-based asteroseismology, ground-based observations should not be ignored. In fact, solar observations have demonstrated that the intrinsic stellar background ‘noise’ from near-surface convection and activity is a much more serious concern in photometric asteroseismic observations than in radial-velocity observations. This is the motivation for the creation of the Danish-led SONG (Stellar Observations Network Group) network of 1 m telescopes dedicated to asteroseismology and exoplanet studies (Grundahl et al. 2014). The first telescope in the network, the Hertzsprung SONG Telescope, is in operation on Tenerife, and the second telescope is in commissioning in Delingha in western China. Funding for further telescopes will be sought from Danish sources and through international collaboration.

Thus, referring again to Eddington, there are excellent prospects to obtain certain, or at least much improved, knowledge of that which is hidden behind the substantial barriers of the stellar surface.

Acknowledgment

I am grateful to the organizers for the invitation to participate in this very interesting conference and the opportunity to visit Azerbaijan. I thank Rasmus Handberg for help with producing Figs 2 and 4, Victor Silva Aguirre for providing Fig. 3 and Benoît Mosser for providing Fig. 6. Funding for the Stellar Astrophysics Centre is provided by The Danish National Research Foundation (Grant DNRF106). The research is supported by the ASTERISK project (ASTERoseismic Investigations with SONG and Kepler) funded by the European Research Council (Grant agreement no.: 267864).

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